7.4 Exercise Set

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Concept Reinforcement *In each of Exercises 1–8, match the expression with an equivalent expression from the column on the right. Assume $a, b > 0$.*

1. (e) $\sqrt[3]{\frac{a^2}{b^6}}$
2. (b) $\frac{\sqrt[3]{a^6}}{\sqrt[3]{b^9}}$
3. (f) $\sqrt[5]{\frac{a^6}{b^4}}$
4. (c) $\sqrt{\frac{a}{b^3}}$
5. (h) $\frac{\sqrt[5]{a^2}}{\sqrt[5]{b^2}}$
6. (d) $\frac{\sqrt{5a^4}}{\sqrt{5a^3}}$
7. (a) $\frac{\sqrt[5]{a^2}}{\sqrt[5]{b^3}}$
8. (g) $\sqrt[4]{\frac{16a^6}{a^2}}$

a) $\frac{\sqrt[5]{a^2}\sqrt[5]{b^2}}{\sqrt[5]{b^5}}$

b) $\frac{a^2}{b^3}$

c) $\sqrt{\frac{a \cdot b}{b^3 \cdot b}}$

d) $\sqrt{a}$

e) $\frac{\sqrt[3]{a^2}}{b^2}$

f) $\sqrt[5]{\frac{a^6 b}{b^4 \cdot b}}$

g) $2a$

h) $\frac{\sqrt[5]{a^2 b^3}}{\sqrt[5]{b^5}}$

Simplify by taking the roots of the numerator and the denominator. Assume that all variables represent positive numbers.

9. $\sqrt{\frac{36}{25}}$ $\frac{6}{5}$
10. $\sqrt{\frac{100}{81}}$ $\frac{10}{9}$
11. $\sqrt[3]{\frac{64}{27}}$ $\frac{4}{3}$
12. $\sqrt[3]{\frac{343}{1000}}$ $\frac{7}{10}$
13. $\sqrt{\frac{49}{y^2}}$ $\frac{7}{y}$
14. $\sqrt{\frac{121}{x^2}}$ $\frac{11}{x}$
15. $\sqrt{\frac{36y^3}{x^4}}$ $\frac{6y\sqrt{y}}{x^2}$
16. $\sqrt{\frac{25a^5}{b^6}}$ $\frac{5a^2\sqrt{a}}{b^3}$
17. $\sqrt[3]{\frac{27a^4}{8b^3}}$ $\frac{3a\sqrt[3]{a}}{2b}$
18. $\sqrt[3]{\frac{64x^7}{216y^6}}$ $\frac{2x^2\sqrt[3]{x}}{3y^2}$
19. $\sqrt[4]{\frac{32a^4}{2b^4c^8}}$ $\frac{2a}{bc^2}$
20. $\sqrt[4]{\frac{81x^4}{y^8z^4}}$ $\frac{3x}{y^2z}$
21. $\sqrt[4]{\frac{a^5b^8}{c^{10}}}$ $\frac{ab^2}{c^2}\sqrt[4]{\frac{a}{c^2}}$
22. $\sqrt[4]{\frac{x^9y^{12}}{z^6}}$ ⊡
23. $\sqrt[5]{\frac{32x^6}{y^{11}}}$ $\frac{2x}{y^2}\sqrt[5]{\frac{x}{y}}$
24. $\sqrt[5]{\frac{243a^9}{b^{13}}}$ ⊡
25. $\sqrt[6]{\frac{x^6y^8}{z^{15}}}$ $\frac{xy}{z^2}\sqrt[6]{\frac{y^2}{z^3}}$
26. $\sqrt[6]{\frac{a^9b^{12}}{c^{13}}}$ ⊡

Divide and, if possible, simplify. Assume that all variables represent positive numbers.

27. $\frac{\sqrt{18y}}{\sqrt{2y}}$ 3
28. $\frac{\sqrt{700x}}{\sqrt{7x}}$ 10
29. $\frac{\sqrt[3]{26}}{\sqrt[3]{13}}$ $\sqrt[3]{2}$
30. $\frac{\sqrt[3]{35}}{\sqrt[3]{5}}$ $\sqrt[3]{7}$
31. $\frac{\sqrt{40xy^3}}{\sqrt{8x}}$ $y\sqrt{5y}$
32. $\frac{\sqrt{56ab^3}}{\sqrt{7a}}$ $2b\sqrt{2b}$
33. $\frac{\sqrt[3]{96a^4b^2}}{\sqrt[3]{12a^2b}}$ $2\sqrt[3]{a^2b}$
34. $\frac{\sqrt[3]{189x^5y^7}}{\sqrt[3]{7x^2y^2}}$ $3xy\sqrt[3]{y^2}$
35. $\frac{\sqrt{100ab}}{5\sqrt{2}}$ $\sqrt{2ab}$
36. $\frac{\sqrt{75ab}}{3\sqrt{3}}$ $\frac{5}{3}\sqrt{ab}$
37. $\frac{\sqrt[4]{48x^9y^{13}}}{\sqrt[4]{3xy^{-2}}}$ $2x^2y^3\sqrt[4]{y^3}$
38. $\frac{\sqrt[5]{64a^{11}b^{28}}}{\sqrt[5]{2ab^{-2}}}$ $2a^2b^6$
39. $\frac{\sqrt[3]{x^3 - y^3}}{\sqrt[3]{x - y}}$ $\sqrt[3]{x^2 + xy + y^2}$
40. $\frac{\sqrt[3]{r^3 + s^3}}{\sqrt[3]{r + s}}$ ⊡

***Hint*: Factor and then simplify.**

Rationalize each denominator. Assume that all variables represent positive numbers.

41. $\sqrt{\frac{3}{2}}$ $\frac{\sqrt{6}}{2}$
42. $\sqrt{\frac{6}{7}}$ $\frac{\sqrt{42}}{7}$
43. $\frac{2\sqrt{5}}{7\sqrt{3}}$ $\frac{2\sqrt{15}}{21}$
44. $\frac{3\sqrt{5}}{2\sqrt{2}}$ $\frac{3\sqrt{10}}{4}$
45. $\sqrt[3]{\frac{5}{4}}$ $\frac{\sqrt[3]{10}}{2}$
46. $\sqrt[3]{\frac{2}{9}}$ $\frac{\sqrt[3]{6}}{3}$
47. $\frac{\sqrt[3]{3a}}{\sqrt[3]{5c}}$ $\frac{\sqrt[3]{75ac^2}}{5c}$
48. $\frac{\sqrt[3]{7x}}{\sqrt[3]{3y}}$ $\frac{\sqrt[3]{63xy^2}}{3y}$

⊡ Answers to Exercises 22, 24, 26, and 40 are on p. IA-16.

19. $\frac{\sqrt[4]{5y^6}}{\sqrt[4]{9x}}$ $\frac{y\sqrt[4]{45x^3y^2}}{3x}$

50. $\frac{\sqrt[5]{3a^4}}{\sqrt[5]{2b^7}}$ $\frac{\sqrt[5]{48a^4b^3}}{2b^2}$

51. $\sqrt[3]{\frac{2}{x^2y}}$ $\frac{\sqrt[3]{2xy^2}}{xy}$

52. $\sqrt[3]{\frac{5}{ab^2}}$ $\frac{\sqrt[3]{5a^2b}}{ab}$

53. $\sqrt{\frac{7a}{18}}$ $\frac{\sqrt{14a}}{6}$

54. $\sqrt{\frac{3x}{10}}$ $\frac{\sqrt{30x}}{10}$

55. $\sqrt{\frac{9}{20x^2y}}$ $\frac{3\sqrt{5y}}{10xy}$

56. $\sqrt{\frac{7}{32a^2b}}$ $\frac{\sqrt{14b}}{8ab}$

Aha! **57.** $\sqrt{\frac{10ab^2}{72a^3b}}$ $\frac{\sqrt{5b}}{6a}$

58. $\sqrt{\frac{21x^2y}{75xy^5}}$ $\frac{\sqrt{7x}}{5y^2}$

Rationalize each numerator. Assume that all variables represent positive numbers.

59. $\sqrt{\frac{5}{11}}$ $\frac{5}{\sqrt{55}}$

60. $\sqrt{\frac{2}{3}}$ $\frac{2}{\sqrt{6}}$

61. $\frac{2\sqrt{6}}{5\sqrt{7}}$ $\frac{12}{5\sqrt{42}}$

62. $\frac{3\sqrt{10}}{2\sqrt{3}}$ $\frac{15}{\sqrt{30}}$

63. $\frac{\sqrt{8}}{2\sqrt{3x}}$ $\frac{2}{\sqrt{6x}}$

64. $\frac{\sqrt{12}}{\sqrt{5y}}$ $\frac{6}{\sqrt{15y}}$

65. $\frac{\sqrt[3]{7}}{\sqrt[3]{2}}$ $\frac{7}{\sqrt[3]{98}}$

66. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$ $\frac{5}{\sqrt[3]{100}}$

67. $\sqrt{\frac{7x}{3y}}$ $\frac{7x}{\sqrt{21xy}}$

68. $\sqrt{\frac{7a}{6b}}$ $\frac{7a}{\sqrt{42ab}}$

69. $\sqrt[3]{\frac{2a^5}{5b}}$ $\frac{2a^2}{\sqrt[3]{20ab}}$

70. $\sqrt[3]{\frac{2a^4}{7b}}$ $\frac{2a^2}{\sqrt[3]{28a^2b}}$

71. $\sqrt{\frac{x^3y}{2}}$ $\frac{x^2y}{\sqrt{2xy}}$

72. $\sqrt{\frac{ab^5}{3}}$ $\frac{ab^3}{\sqrt{3ab}}$

TW **73.** Explain why it is easier to approximate

$$\frac{\sqrt{2}}{2} \text{ than } \frac{1}{\sqrt{2}}$$

if no calculator is available and we know that $\sqrt{2} \approx 1.414213562$.

TW **74.** A student *incorrectly* claims that

$$\frac{5+\sqrt{2}}{\sqrt{18}} = \frac{5+\sqrt{1}}{\sqrt{9}} = \frac{5+1}{3}.$$

How could you convince the student that a mistake has been made? How would you explain the correct way of rationalizing the denominator?

SKILL REVIEW

To prepare for Section 7.5, review factoring expressions and multiplying polynomials (Sections 5.2 and 5.3).

Factor. [5.3]

75. $3x - 8xy + 2xz$ $x(3 - 8y + 2z)$

76. $4a^2c + 9ac - 3a^3c$ $ac(4a + 9 - 3a^2)$

Multiply. [5.2]

77. $(a + b)(a - b)$ $a^2 - b^2$

78. $(a^2 - 2y)(a^2 + 2y)$ $a^4 - 4y^2$

79. $(8 + 3x)(7 - 4x)$ $56 - 11x - 12x^2$

80. $(2y - x)(3a - c)$ $6ay - 2cy - 3ax + cx$

SYNTHESIS

TW **81.** Is the quotient of two irrational numbers always an irrational number? Why or why not?

TW **82.** Is it possible to understand how to rationalize a denominator without knowing how to multiply rational expressions? Why or why not?

83. *Pendulums.* The *period* of a pendulum is the time it takes to complete one cycle, swinging to and fro. For a pendulum that is L centimeters long, the period T is given by the formula

$$T = 2\pi\sqrt{\frac{L}{980}},$$

where T is in seconds. Find, to the nearest hundredth of a second, the period of a pendulum of length **(a)** 65 cm; **(b)** 98 cm; **(c)** 120 cm. Use a calculator's π key if possible. **(a)** 1.62 sec; **(b)** 1.99 sec; **(c)** 2.20 sec

Perform the indicated operations.

84. $\frac{7\sqrt{a^2b}\,\sqrt{25xy}}{5\sqrt{a^{-4}b^{-1}}\,\sqrt{49x^{-1}y^{-3}}}$ a^3bxy^2

85. $\frac{\left(\sqrt[3]{81mn^2}\right)^2}{\left(\sqrt[3]{mn}\right)^2}$ $9\sqrt[3]{9n^2}$

86. $\frac{\sqrt{44x^2y^9z}\,\sqrt{22y^9z^6}}{\left(\sqrt{11xy^8z^2}\right)^2}$ $2yz\sqrt{2z}$